Self-similar pulses in coherent linear amplifiers

Soodeh Haghgoo and Sergey A. Ponomarenko*

Department of Electrical and Computer Engineering, Dalhousie University, Halifax, NS, B3J 2X4, Canada

**serpo@dal.ca*

Abstract: We discover and analytically describe self-similar pulses existing in homogeneously broadened amplifying linear media in a vicinity of an optical resonance. We demonstrate numerically that the discovered pulses serve as universal self-similar asymptotics of any near-resonant short pulses with sharp leading edges, propagating in coherent linear amplifiers. We show that broadening of any low-intensity seed pulse in the amplifier has a diffusive nature: Asymptotically the pulse width growth is governed by the simple diffusion law. We also compare the energy gain factors of short and long self-similar pulses supported by such media.

OCIS codes: (320.0320) Ultrafast optics; (320.5550) Pulses; (320.5540) Pulse shaping; (260.0260) Physical optics; (260.2030) Dispersion.

References and links

1. J. M. Dudley, C. Finot, D. J. Richardson, and G. Millot, "Self-similarity in ultrafast nonlinear optics," Nat. Phys. **3**, 597–603 (2007).
2. S. An and J. E. Sipe, "Universality in the dynamics of phase grating formation in optical fibers," Opt. Lett. **16**, 1478–1480 (1991).
3. C. R. Menyuk, D. Levi, and P. Winternitz, "Self-similarity in transient stimulated Raman scattering," Phys. Rev. Lett. **69**, 3048–3051 (1992).
4. T. M. Monroe, P. D. Millar, P. L. Poladian, and C. M. de Sterke, "Self-similar evolution of self-written waveguides," Opt. Lett. **23**, 268–270 (1998).
5. M. Soljacic, M. Segev, and C. R. Menyuk, "Self-similarity and fractals in soliton-supporting systems," Phys. Rev. E **61**, R1048–R1051 (2000).
6. D. Anderson, M. Desaix, and M. Karlsson, M. Lisak, and M. L. Quiroga-Teixeiro, "Wave-breaking-free pulses in nonlinear optical fibers," J. Opt. Soc. Am. B **10**, 1185–1190 (1993).
7. K. Tamura and M. Nakazawa, "Pulse compression by nonlinear pulse evolution with reduced optical wave breaking in erbium-doped fiber amplifiers," Opt. Lett. **21**, 68–70 (1996).
8. M. E. Fermann, V. I. Kruglov, B. C. Thomsen, J. D. Dudley, and J. D. Harvey, "Self-similar propagation and amplification of parabolic pulses in optical fibers," Phys. Rev. Lett. **84**, 6010–6013 (2000).
9. F. O. Ilday, J. R. Buckley, W. G. Clark, and F. W. Wise, "Self-similar evolution of parabolic pulses in a laser," Phys. Rev. Lett. **92**, 213902 (2004).
10. B. Oktem, F. O. Ulgudur, and F. O. Ilday, "Soliton-similariton fibre laser," Nat. Photon. **4**, 307–311 (2010).
11. W. H. Renninger, A. Chong, and F. W. Wise, "Self-similar pulse evolution in an all-normal-dispersion laser," Phys. Rev. A **82**, 021805(R) (2010).
12. S. A. Ponomarenko and G. P. Agrawal, "Do soliton-like self-similar waves exist in nonlinear optical media?" Phys. Rev. Lett. **97**, 013901 (2006).
13. S. A. Ponomarenko and G. P. Agrawal, "Optical similaritons in nonlinear waveguides," Opt. Lett. **32**, 1659–1661 (2007).
14. L. Wu, J.-F. Zhang, L. Li, and Q. Tian, "Similaritons in nonlinear optical systems," Opt. Express **16**, 6352–6360 (2008).
15. S. A. Ponomarenko and S. Haghgoo, "Spatial optical similaritons in conservative nonintegrable systems," Phys. Rev. A **81**, 051801(R) (2010).

#144374 - $15.00 USD Received 18 Mar 2011; revised 21 Apr 2011; accepted 21 Apr 2011; published 4 May 2011

16. K. D. Moll, A. Gaeta, and G. Fibich, "Self-similar optical wave collapse: observation of the townes profile," Phys. Rev. Lett. **90**, 203902 (2003).
17. S. V. Manakov, "Propagation of an ultrashort optical pulse in a two-level laser amplifier," Sov. Phys. JETP **56**, 37–44 (1982).
18. I. R. Gabitov and S. V. Manakov, "Propagation of ultrashort pulses in degenerate laser amplifiers," Phys. Rev. Lett. **50**, 495–498 (1983).
19. I. R. Gabitov, V. E. Zakharov, and A. V. Mikhailov, "Nonlinear theory of superfluorescence," Sov. Phys. JETP **59**, 703–709 (1984).
20. S. A. Ponomarenko and S. Haghgoo, "Self-similarity and optical kinks in resonant nonlinear media," Phys. Rev. A **82**, 051801(R) (2010).
21. L. Allen and J. H. Eberly, *Optical Resonance and Two-Level Atoms* (Dover Publications Inc., 1975).
22. P. W. Milonni and J. H. Eberly, *Lasers* (Wiley, 1985).
23. V. V. Kozlov and S. Wabnitz, "Quasi-parabolic pulses in a coherent nonlinear optical amplifier," Opt. Lett. **35**, 2058–2060 (2010).
24. A small deviation from the universal asymptotics in the pulse tails can be explained by limited accuracy of our sharp leading edge approximation.
25. M. Abramowitz and I. A. Stegan, *Handbook of Mathematical Functions* (Dover, 1972).

1. Introduction

Only not too long ago did the optical community realize [1] that self-similarity is quite a ubiquitous feature of optical systems. The phenomena as diverse as optical pulse evolution in Hall gratings [2], stimulated Raman scattering [3], formation of self-written waveguides [4], and fractal formation in nonlinear media [5] exhibit self-similarity in one form or another. Recently, long-term self-similar evolution of pulses in nonlinear fiber amplifiers [6–8], in passive fibers of lasers [9] has received much attention due to its fundamental interest and potential applications. Lately, fiber lasers with self-similar evolution in the amplifier and soliton evolution in the anomalous dispersion segments [10] as well as all-normal-dispersion lasers working in a self-similar light propagation regime [11] were proposed and experimentally realized. Self-similar dynamics of beams in nonlinear waveguide amplifiers and in conservative nonlinear media have also been explored [12–16].

Self-similar evolution of pulses and beams in resonant media has also been explored. In particular, universal quasi-self-similar asymptotics of ultrashort light propagation in coherent nondegenerate and degenerate nonlinear amplifiers was examined in Refs. [17] and [18], respectively. Also, self-similarity in superfluorescence in homogeneously broadened resonant media was explored as well [19]. In addition to the early pioneering work [17–19], however, some recent studies [15,20] show that a wealth of self-similar regimes exists in such media. In particular, self-similar beams can be generated in cubic-quintic nonlinear media doped with resonant impurities in the limit of a large detuning from the impurity resonance [15]. By the same token, we have shown elsewhere [20] that in resonant nonlinear absorbers, self-similar optical kinks are formed as intermediate asymptotics of any incident pulse with a long tail in the trailing edge. In this context, it is instructive to explore the possibility of self-similar pulse formation in *resonant linear* media. At first glance, the very proximity to optical resonance(s), coupled with the system linearity, appears to preclude self-similarity of a sufficiently short pulse: Strong dispersion at resonance(s) would, in general, seem to cause severe pulse reshaping. One would then also wonder whether self-similar pulses in such media, if any, would be universal asymptotics of very weak seed pulses. The affirmative answer to the last question would augur well for the experimental realization of such similaritons.

In this paper, we demonstrate analytically that self-similar optical pulses, albeit of a highly asymmetric shape, can indeed propagate in resonant linear amplifiers. Such an asymmetric self-similar shape is a manifestation of dynamic balance between linear amplification and phase relaxation processes in resonant propagation of short pulses in the absence of inhomogeneous broadening and host medium dispersion. We further show that a low-intensity seed pulse of any

#144374 - $15.00 USD Received 18 Mar 2011; revised 21 Apr 2011; accepted 21 Apr 2011; published 4 May 2011

profile with a sharp leading edge evolves into a self-similar one upon propagation inside the amplifier. The short pulse broadening has a universal diffusive character such that the rms width grows as a square root of the propagation distance. Thus, we demonstrate, both analytically and with numerical simulations that the discovered self-similar pulses are universal intermediate asymptotics in resonant coherent amplifiers. The intermediate character of the asymptotics is imposed by the system linearity: As long as the pulse area will have grown enough, our linear approximation surely breaks down; sufficiently small initial pulse areas and/or short enough amplifier lengths are required for the linear approximation to hold over the entire amplifier length.

2. Physical model and mathematical preliminaries

We model a resonant medium as a collection of two-level atoms with the resonance frequency ω_0, thereby limiting our consideration to the case of one internal resonance. We assume hereafter that the pulse spectrum is mainly affected by homogeneous broadening, implying that $\gamma_\perp \gg \delta$, $\gamma_\perp$ and δ being transverse (dipole moment) relaxation rate and a characteristic spectral width of inhomogeneous broadening, respectively. Under these conditions, the evolution of a pulse with the carrier frequency ω in the medium is governed by a reduced wave equation,

$$\partial_\zeta \Omega = i\kappa\sigma; \tag{1}$$

subject to the slowly-varying envelope approximation (SVEA):

$$\partial_\zeta \Omega \ll k\Omega, \qquad \partial_\tau \Omega \ll \omega\Omega. \tag{2}$$

Here $\Omega = 2d_{eg}\mathscr{E}/\hbar$ is the Rabi frequency associated with the pulse amplitude $\mathscr{E}$, N is an atom density, d_{eg} is a dipole matrix element between the ground and excited states of any atom, labeled with the indices g and e, respectively; $\kappa = \omega N|d_{eg}|^2/c\varepsilon_0\hbar$ is a coupling constant, $k = \omega/c$, and Eq. (1) is written in terms of the transformed coordinate and time, $\zeta = z$ and $\tau = t - z/c$. The dipole moment matrix element σ and one-atom inversion w obey the Bloch equations [21]

$$\partial_\tau \sigma = -(\gamma_\perp + i\Delta)\sigma - i\Omega w, \tag{3}$$

and

$$\partial_\tau w = -\gamma_\parallel (w - w_{eq}) - \tfrac{i}{2}(\Omega^*\sigma - \Omega\sigma^*). \tag{4}$$

In Eqs. (3) and (4) $\gamma_\parallel$ is a longitudinal relaxation rates associated with one-atom inversion damping, $\Delta = \omega - \omega_0$ is a detuning from the resonance and w_{eq} is a value of the one-atom inversion in the absence of the pulse (equilibrium).

In the low-intensity limit, the atomic population is hardly affected by the pulse such that the one-atom inversion is approximately given by its equilibrium value,

$$w \simeq w_{eq} = \pm 1, \tag{5}$$

where the upper/lower sign corresponds to amplifier/absorber case: All atoms remain in the upper/lower level. Physically, the linear amplification regime of a weak probe pulse can be realized using a strong pump in a three-level configuration, standard of laser systems, see e.g. [22]. The latter is illustrated schematically in Fig. 1 where the pump rate P and the upper level relaxation rate must be large enough, $P \gg \gamma_\perp$ and $\Gamma \gg \gamma_\perp$, to achieve population inversion between levels “e” and “g”.

Mathematically, the approximation (5) implies linearization of the dipole moment evolution equation viz.,

$$\partial_\tau \sigma = -(\gamma_\perp + i\Delta)\sigma \mp i\Omega. \tag{6}$$

#144374 - $15.00 USD Received 18 Mar 2011; revised 21 Apr 2011; accepted 21 Apr 2011; published 4 May 2011

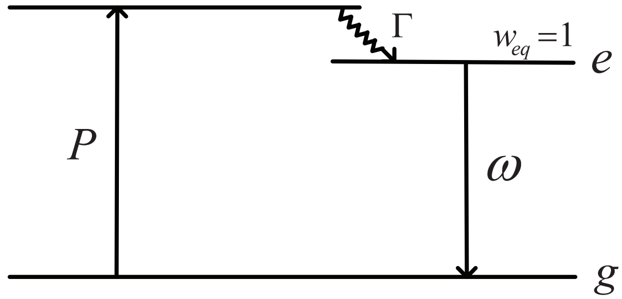

Fig. 1. Schematics of a pump-probe three-level system modeling coherent linear amplifier. The resonant transition takes place between levels e and g.

In the following, we distinguish two limiting cases: "long" pulses, $t_p \gg T_\perp$ and "short" ones, $t_p \leq T_\perp$–in the case $t_p \ll T_\perp$, the pulses may be called "ultrashort"–where $T_\perp = \gamma_\perp^{-1}$ and t_p is a characteristic pulse width.

Long pulses. In this case, the atomic variables can be adiabatically eliminated–using Eq. (6) and equating σ to its quasi-steady-state value with respect to Ω–which will result in the pulse evolution equation in the form

$$\partial_\zeta \Omega = \pm \left(\frac{\alpha + i\beta}{2}\right)\Omega. \tag{7}$$

Here we introduced an inverse Beer's gain/absorption length α and an overall phase accumulation rate β by the expressions,

$$\alpha = \frac{2\kappa\gamma_\perp}{\gamma_\perp^2 + \Delta^2}, \qquad \beta = \frac{2\kappa\Delta}{\gamma_\perp^2 + \Delta^2}. \tag{8}$$

It follows at once from Eq. (7) that for sufficiently long pulses, *any* pulse grows/decays exponentially in such a medium, maintaining its overall shape,

$$\Omega(\tau, \zeta) = \Omega_0(\tau) e^{\pm(\alpha + i\beta)\zeta/2}, \tag{9}$$

where $\Omega_0(t)$ describes an initial pulse profile, and Eq. (9) is well-known Beer's amplification/absorption law.

Short pulses. For simplicity, we consider pulses exactly on resonance with the atomic transition, $\Delta = 0$; the pulse field and dipole moment evolution equations can then be written as

$$\partial_\zeta \Omega = \frac{i}{2}\gamma_\perp \alpha_0 \sigma, \tag{10}$$

and

$$\partial_\tau \sigma = -\gamma_\perp \sigma \mp i\Omega. \tag{11}$$

where $\alpha_0 = 2\kappa/\gamma_\perp$. Our treatment to this point is equally applicable to amplifying and absorbing media. Hereafter we focus on short pulse propagation in amplifiers.

3. Short self-similar pulses

The inspection of Eqs. (10) and (11) reveals that the electric field of a self-similar pulse and atomic dipole moment profiles ought to be sought in the form

$$\Omega(\zeta, \tau) = \gamma_\perp \theta(\tau) \overline{\Omega}[\gamma_\perp \tau F(\zeta)] e^{-\gamma_\perp \tau}, \tag{12}$$

#144374 - $15.00 USD Received 18 Mar 2011; revised 21 Apr 2011; accepted 21 Apr 2011; published 4 May 2011

and

$$\sigma(\zeta,\tau)=\theta(\tau)G(\zeta)\overline{\sigma}[\gamma_{\perp}\tau F(\zeta)]e^{-\gamma_{\perp}\tau}. \tag{13}$$

Here $\theta(\tau)$ is a unit step function describing a sharp leading edge of the pulse, $F(\zeta)$ and $G(\zeta)$ are arbitrary at the moment and $\overline{\Omega}$ and $\overline{\sigma}$ are dimensionless functions. Substituting the Ansatz (12) and (13) into Eqs. (10) and (11), we can show that self-similarity is sustained provided that

$$F(\zeta)=\alpha_0\zeta+T_{\perp}/t_p; \qquad G(\zeta)=1/F(\zeta). \tag{14}$$

Further, the dimensionless pulse envelope $\overline{\Omega}$ in the amplifying medium satisfies the equation,

$$\eta\overline{\Omega}''+\overline{\Omega}'-\overline{\Omega}/2=0, \tag{15}$$

where we introduced a similarity variable η by

$$\eta=\gamma_{\perp}\tau(\alpha_0\zeta+T_{\perp}/t_p). \tag{16}$$

and the prime denotes a derivative with respect to η.

Analytically solving Eq. (15), we can obtain a self-similar pulse envelope in a linear amplifier. The overall pulse profile can then be represented as

$$\Omega(\eta,\tau)\propto\gamma_{\perp}\theta(\tau)\,{}_1F_1(1/2,1,-2\sqrt{2\eta})\exp(\sqrt{2\eta}-\gamma_{\perp}\tau), \tag{17}$$

where ${}_1F_1(a,c,x)$ is a confluent hypergeometric function, and we dropped an arbitrary (small) initial pulse amplitude. Eq. (17) can be expressed in a more compact form as

$$\Omega(\eta,\tau)\propto\gamma_{\perp}\theta(\tau)I_0(\sqrt{2\eta})\exp(-\gamma_{\perp}\tau), \tag{18}$$

where $I_0(x)$ is a modified Bessel function of zero order. We note that for sufficiently long propagation distances, $\alpha_0\zeta\gg T_{\perp}/t_p$, the self-similar pulse profile no longer depends on t_p, yielding a universal self-similar profile

$$\Omega(\tau,\zeta)\propto\gamma_{\perp}\theta(\tau)I_0(2\sqrt{\kappa\zeta\tau})\exp(-\gamma_{\perp}\tau). \tag{19}$$

It can be inferred from the analysis of Eq. (18) that the pulse evolution is governed by a synergy of three factors: pulse shape asymmetry, coherent gain and dipole phase relaxation. In the absence of nonlinearity, the self-similarity arises as a consequence of dynamic balance between coherent gain and linear damping; the sharp leading edge of the pulse profile ensures the balance in the absence of bulk medium dispersion and inhomogeneous broadening. Thus, the asymmetry of a seed pulse shape appears to be the only requirement for self-similarity in the studied linear system to emerge.

We stress here that in the linear limit, damping of the trailing edge of the pulse by the linear relaxation processes allows for the finite energy self-similar pulse formation. The situation here is drastically different from quasi-self-similarity emerging in the nonlinear amplification of ultrashort pulses–the term should be understood in the sense defined in Sec. 2–studied in [17]. In the latter case, linear damping is negligible and the nonlinearity promotes the emergence of finite-energy pulses in the amplifying medium. We notice also that the discovered self-similar pulses have no chirp–since dispersion plays no role here–which sets them apart from more familiar parabolic pulses in nonlinear fiber amplifiers. The latter require a linear chirp to prevent wave breaking [6,8]. We also note that owning to a different physical nature, our similaritons are markedly different from recently discovered quasi-parabolic pulses in nonlinear amplifiers [23]. While the former are chirp-free self-similar pulses, the latter are phase-modulated steady-state pulses moving with the speed of light.

#144374 - $15.00 USD Received 18 Mar 2011; revised 21 Apr 2011; accepted 21 Apr 2011; published 4 May 2011

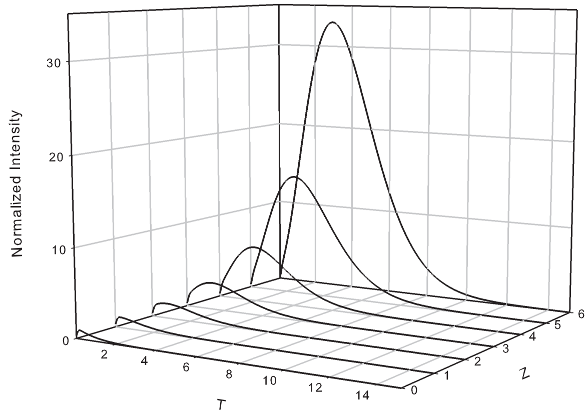

Fig. 2. Normalized intensity of a short self-similar pulse as a function of dimensionless time $T = \tau/T_{\perp}$ and propagation distance $Z = \alpha_0\zeta$. The pulse intensity is normalized to its peak value at $Z = 0$. The initial pulse width is chosen to be $t_p = T_{\perp}$.

We now proceed to describing the properties of new self-similar pulses. In Fig. 2, we display the self-similar pulse profile evolution as a function of the dimensionless time $T = \tau/T_{\perp}$ for several values of the dimensionless propagation distance $Z = \alpha_0\zeta$. The self-similar character of the pulse dynamics is clearly discernable in the figure. To demonstrate the universal nature of the discovered self-similar regime, we numerically simulate the evolution of a generic asymmetric Gaussian pulse, $\Omega_1(t,0) \propto \theta(t)\exp(-t^2/t_p^2)$, in the amplifier and compare its profile with the self-similar asymptotics. The results are presented in Figure 3. To ensure the two pulses are sufficiently different in the source plane, we take the Gaussian pulse to be half as long as the self-similar one at $Z = 0$: $t_p = T_{\perp}/2$. In the inset to the figure, we compare short-distance pulse dynamics of the two pulses. We see in the figure that although the Gaussian pulse profile deviates from the self-similar asymptotics over short distances–at least over first few Beer's amplification lengths as is seen in the inset–it quickly converges to the universal asymptotics over longer distances. It then is seen to coincide with the self-similar asymptotics profile to within numerical round-off errors [24]. We obtained qualitatively similar results for hyperbolic secant and exponential profiles with cut off leading edges: $\Omega_2(t,0) \propto \theta(t)\mathrm{sech}(t/t_p)$, and $\Omega_3(t,0) \propto \theta(t)\exp(-t/t_p)$.

To reenforce the message, we examine the rms width– defined as $\Delta T = \sqrt{\langle T^2\rangle - \langle T\rangle^2}$ and measured in the units of $T_{\perp}$–of the universal self-similar asymptotics on pulse propagation in the amplifier. The averaging is taken over the pulse intensity distribution, for instance,

$$\langle T^2(Z)\rangle \equiv \frac{\int_0^\infty dT\,T^2\,|\mathscr{E}(T,Z)|^2}{\int_0^\infty dT\,|\mathscr{E}(T,Z)|^2}. \tag{20}$$

With the help of the asymptotic expansion of $I_0(x)$ [25], an analytical expression for the rms width can be derived and presented in an exceptionally simple form

$$\Delta T(Z) \simeq \sqrt{3Z}/2. \tag{21}$$

In other words, the pulse rms width grows with the distance in a diffusive manner with the effective diffusion coefficient equal to $3\alpha T_{\perp}^2/8$ (in original units). We then evaluate and display

#144374 - $15.00 USD Received 18 Mar 2011; revised 21 Apr 2011; accepted 21 Apr 2011; published 4 May 2011

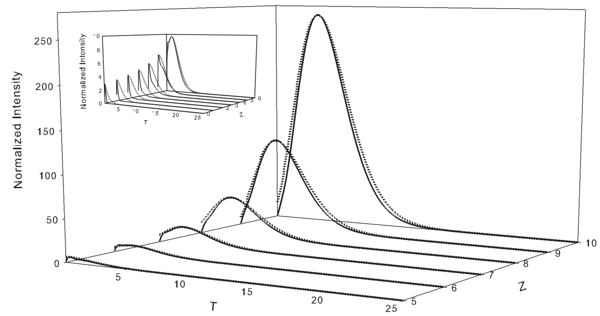

Fig. 3. Normalized intensity of a short Gaussian (solid) and self-similar (dashed) pulses as functions of dimensionless time $T = \tau/T_{\perp}$ and propagation distance $Z = \alpha_0\zeta$. The pulse intensities are normalized to their peak values at $Z = 0$. The initial pulse width is chosen to be $t_p = T_{\perp}/2$ and $t_p = T_{\perp}$ for Gaussian and self-similar pulses, respectively. The inset shows pulse dynamics for short propagation distances.

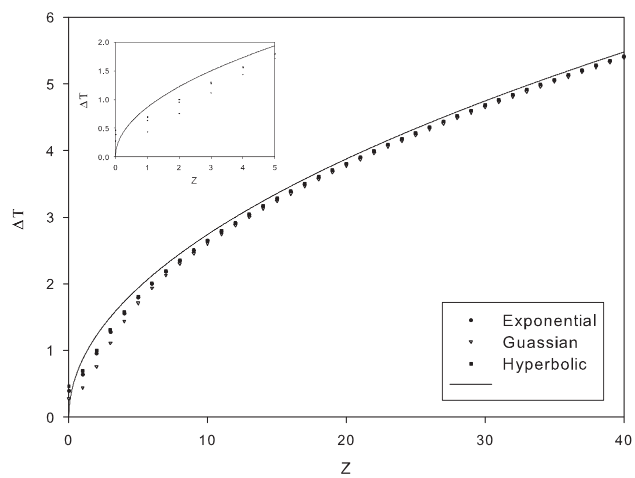

Fig. 4. Average widths of Gaussian, secant hyperbolic and exponential pulses as functions of the dimensionless propagation distance $Z = \alpha_0\zeta$. The self-similar asymptotic pulse width dependence on the propagation distance is shown as the solid curve.

the behavior of asymmetric Gaussian, hyperbolic secant, and exponential pulse widths in Fig. 4. The self-similar asymptotic pulse width is drawn in a solid curve. In the inset to the figure, we exhibit the pulse width dynamics over a short range of propagation distances. We can conclude from the figure that although the width of an arbitrarily shaped seed pulse initially deviates from the self-similar pulse width, the former asymptotically tends to the latter over a long enough propagation distance, thereby underscoring the universal character of the discovered self-similar asymptotics.

Next, we observe that the applicability of SVEA is not, in general, guaranteed for pulses with sharp fronts. Hence, the presence of a step function has to be physically justified as follows. A practical realization of an ideal sawtooth-like pulse involves a finite switching time t_{sw} describing the fast rise of its leading edge. Thus for a short pulse, $t_p \sim T_{\perp}$, the pulse duration (rise time

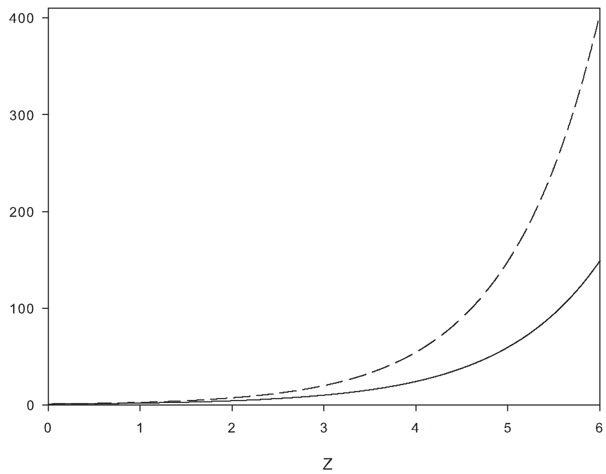

Fig. 5. Energy gain factor $G(Z)$ for a short (solid) and long (dashed) self-similar pulse as a function of the dimensionless propagation distance $Z = \alpha_0\zeta$.

of the pulse front edge) has to be much shorter than the pulse width, yet much longer than an optical cycle for the SVEA–see Eq. (2)–to hold:

$$\omega^{-1} \ll t_{sw} \ll t_p \sim T_\perp. \tag{22}$$

The complimentary conditions (22) can be realized in a laboratory for picosecond pulses in dilute atomic vapors, say, for which, typically $T_\perp \sim 1 \div 10$ ps [21] by choosing, for example, $t_{sw} \sim 10 \div 100$ fs. Mathematically, the leading front step function can then be approximated, for instance, as

$$\theta(\tau) \simeq [1 + \tanh(\tau/t_{sw})]/2, \tag{23}$$

with the excellent approximation attainable for $t_{sw} = 0.01T_\perp$.

Finally, we exhibit in Fig. 5 a short-pulse energy gain factor,

$$G(\zeta) = \int d\tau |\mathscr{E}(\zeta,\tau)|^2 / \int d\tau |\mathscr{E}(0,\tau)|^2, \tag{24}$$

for the novel self-similar pulses as a function of the propagation distance. The exponential gain factor for long pulses,

$$G_0(\zeta) = \exp(\alpha_0\zeta), \tag{25}$$

is presented for comparison as well. On comparing the two, we conclude that for sufficiently long distances, long pulses are amplified much more efficiently than are short ones. This is because short pulses have very broad energy spectra with large fractions of their energies stored in the pulse tails. The latter lie well outside of the medium gain spectrum and are then not efficiently amplified. Narrow spectra of long pulses, on the other hand, fall entirely within the medium gain spectrum, which results in strong amplification. These qualitative conclusions are bourn out by the asymptotic analysis yielding the following universal long-term gain behavior for asymmetric short pulses

$$G_\infty(\zeta) \propto \frac{e^{\alpha_0\zeta}}{\sqrt{\alpha_0\zeta}}. \tag{26}$$

#144374 - $15.00 USD Received 18 Mar 2011; revised 21 Apr 2011; accepted 21 Apr 2011; published 4 May 2011

Hence, comparing Eqs. (26) and (25), we see that the long-pulse gain dwarfs the short-pulse one in the long-term limit.

In summary, we have discovered a self-similar regime of short pulse propagation in linear amplifiers in the vicinity of an optical resonance. The novel self-similar pulses have sharp leading front, resulting in a highly asymmetric sawtooth-like pulse profile. We have shown that the new pulses serve as intermediate universal asymptotics for any asymmetrically shaped pulse propagation in resonant amplifiers in the linear regime. We note that our results hold true in the absence of inhomogeneous broadening. It will be instructive to determine the influence of the latter on the emergence of universal self-similar asymptotics in the system. This topic will be addressed in detail in a forthcoming publication.

#144374 - $15.00 USD Received 18 Mar 2011; revised 21 Apr 2011; accepted 21 Apr 2011; published 4 May 2011